Tutorial 2.

Preliminary:

① Nominal Interest Rate VS. Effective Interest Rate:

$$1+i = \left[1+\frac{i^{(m)}}{m}\right]^m \Rightarrow i^{(m)} = m\left[(1+i)^{\frac{1}{m}}-1\right], \quad \lim_{m\to\infty} i^{(m)} = \ln(1+i).$$

② Effective Annual Rate of Discount:

$$d = \frac{A(1)-A(0)}{A(1)} \Rightarrow A(0) = A(1)(1-d) \Rightarrow A(n) = A(0)(1-d)^{-n}.$$

~~[illegible]~~ with $A(0) = A(1)v$, $\Rightarrow 1-d = v = \frac{1}{1+i} \Rightarrow d = \frac{i}{1+i}$, or $i = \frac{d}{1-d}$.

③ The Force of Interest:

$\delta_t = \frac{A'(t)}{A(t)}$, i) simple interest rate: $A(t) = A(0)[1+it]$, $A'(t) = A(0)\cdot i$, then $\delta_t = \frac{A'(t)}{A(t)} = \frac{i}{1+it}$.

ii) compound rate: $A(t) = A(0)(1+i)^t$, $A'(t) = A(0)(1+i)^t \cdot \ln(1+i)$, then $\delta_t = \ln(1+i)$

$1+i = e^{\delta_t}$

④ Inflation:

The real rate with interest rate i and inflation rate r.

$$i_{real} = \frac{i-r}{1+r}.$$

1.4.5.

0	3	6	9	12 months
1	$0.99(1+\frac{i^{(4)}}{4})$	$[0.99(1+\frac{i^{(4)}}{4})]^2$		$(0.99(1+\frac{i^{(4)}}{4}))^4$

$i^{(4)} = 3.25\%$, $A(0) = 1$,

$A(12) = \left(0.99\left(1+\frac{i^{(4)}}{4}\right)\right)^4 = 0.992198.$

effective rate $= \frac{A(12)-A(0)}{A(0)} = -0.0078.$

1.5.5.

$A(n) = A(0)(1-d)^{-n}$, $A_B(0) = 100$, $A_R(0) = 50$.

$A_B(11) - A_B(10) = A_R(17) - A_R(16) = X.$

$100[(1-d)^{-11} - (1-d)^{-10}] = 50[(1-d)^{-17} - (1-d)^{-16}]$

$\Rightarrow 100(1-d)^{-10} = 50(1-d)^{-16} \Rightarrow d = 0.1091$

$X = 38.9.$

1.6.6

Bruce: $A(t) = \cancel{A(0)\left[\left(1+\frac{i^{(m)}}{m}\right)^m\right]}$ $A(0)(1+i)^t = A(0)\left[\left(1+\frac{i^{(m)}}{m}\right)^m\right]^t$.

year

$200 = 100\left[\left(1+\frac{i}{2}\right)^2\right]^{7.25} = 100\left(1+\frac{i}{2}\right)^{14.5} = 200$.

$i = 0.0979$.

Peter: $A(t) = \cancel{A(0)e^{\delta t}} \Rightarrow 200 = 100\, e^{7.25\delta} \Rightarrow \delta = 0.0956. \Rightarrow i - \delta = 0.23\%$.

$A(0)(1+i)^t = A(0)\, e^{\delta t}$

$(i^{(m)} > i^{(\infty)})$.

1.7.2.

$i_{real} = \frac{i-r}{1+r}$, $i = 0.12*(1-0.45)$, $r = 0.1$, $i_{real} = \frac{0.12*0.55-0.1}{1+0.1} = -0.0309$.

after tax

1.4.5

1.5-5S

1.6.6S

1.7.2

inflation

1.4.1

$i^{(m)}$: nominal annual, effective annual rate: $i = [1+\frac{i^{(m)}}{m}]^m - 1$.

1.4.2.

(a) $v = \frac{1}{1+\frac{i^{(2)}}{2}}$, $t = 10\times 2 = 20$. $1000 v_{0.095}^{20}$

(b) $v = \frac{1}{1+\frac{i^{(6)}}{6}}$, $t = 10\times 6 = 60$. $1000 v_{0.05}^{60}$

(c) $v = \frac{1}{1+\frac{i^{(12)}}{12}}$, $t = 10\times 12 = 120$. $1000 v_{0.0075}^{120} = 407.94$

1.4.3.

effective annual rate: $i = [1+\frac{0.15}{2}]^2 - 1 = 0.156$ M Bank

R bank: $(1+\frac{i^{(365)}}{365})^{365} \geq 0.156$ ← annual

$i^{(365)} \geq 0.145$.

1.46 $i = [1+\frac{i^{(m)}}{m}]^m - 1 \Rightarrow i^{(m)} = m[(1+i)^{\frac{1}{m}} - 1]$

$m = 0.5, 0.25, 0.1, 0.01$

1.4.7.

$i = 11.25\%$, $\frac{2}{12}(0.1125)\times 1000 = 18.75$.

$\frac{18.75}{1000} = 0.01875$. ~~effec~~ effective rate

Nov 9th → Jan 1st. 53 days. $i[1+0.01875)^{\frac{365}{53}} - 1 = 0.1365$

1.5.1.

(a) $d = \frac{A(1)-A(0)}{A(1)}$, $A(0) = A(1)(1-d)$, $1-d = (1-\frac{d^{(m)}}{m})^m$. $v = 1-d$.

$i = 8\%$. $X = 4992(1+i)^{\frac{1}{2}}$.

(b) $X = 4992(1+i\times\frac{1}{2})$

(c) $d = 8\%$. $\frac{4992}{(1-8\%)^{\frac{1}{2}}}$

(d) $\frac{4992}{(1-8\%\times\frac{1}{2})}$

1.5.4.

$1.3(1-d)=1.15.$

1.5.6.

$A(n)=X$, $A(0)=1$.

$\frac{A(0)}{A(n)}=(1-d)^n \Rightarrow X=(1-d)^{-n}.$

1.5.8.

(a) $\frac{A(0)}{A(t)}=1-dt=\frac{1}{1+it} \Rightarrow 1-d\frac{n}{365}=1+\frac{1}{1+i\cdot\frac{n}{365}} \Rightarrow i=\frac{d}{1-d\frac{n}{365}}$. n1

(b) $1-dt=\frac{1}{1+it} \Rightarrow d=\frac{i}{1+it}$ $i=11\%$, $t=1$, $t=\frac{1}{2}$ $t=\frac{1}{12}$.

1.5.11.

$1200(1-d)=1000(1+i)$, $d=i$,

$i=9.09\%$.

$\frac{A(0)}{A(t)}=\frac{1}{1+it}=1-dt$

$t=\frac{n}{365}$.

1.7.1.

annual rate i, inflation rate r,

$i_{real}=\frac{i-r}{1+r}=\frac{0.1-0.15}{1+0.15}=-0.043$

1.7.4.

pay for bank $100,000\times(1+i)=100,000\times1.1=110,000.$

selling price $100,000\times(1+0.15)=115,000.$ gain $=5,000.$

1.7.6.

$i_{real}=\frac{i-r}{1+r}$. $\frac{18\%-14\%}{1+14\%}=\frac{i-100\%}{1+100\%} \Rightarrow 107\%.$

1.6.1, 1.6.2, 1.6.4, 1.6.7, 1.6.9, 1.7.1, 1.7.4, 1.7.6.

1.6.1.

$A(n)=A(0)e^{\int_0^n \delta_t dt}$, $A(1)=10,000\,e^{0.05}=10,512.71.$

$A(2)=10,000\,e^{0.05+\int_1^2 0.05+0.02(t-1)\,dt}=11,162.78$

1.6.2.

$i^{(4)}$? ~~ee~~ effective annual rate $i: (\frac{i^{(4)}}{4}+1)^{4\cdot\frac{4}{12}}=e^{\int_0^3 0.025\,dt+\int_3^4 0.045\,dt} \Rightarrow i^{(4)}=0.0339$

1.6.4.

Tawny: $(1+\frac{10\%}{2})^2-1$ is effect annual rate.

Fabio's Force of interest: $\delta=\frac{i}{1+it}=\frac{i}{1+5i}$

$Z=1000[1+5i]=1953$

compound interest at annual rate i

Force of Interest $\delta_t=\ln(1+i)$. $=0.09758$

$\frac{i}{1+5i}=0.09758 \Rightarrow$ $i=0.1906$.

1.6.7.

$1000e^{\delta}(e^{1.5\delta})^4=1360.86 \Rightarrow e^{\delta}=1.045$, then $1+i=e^{\delta} \Rightarrow i=e^{\delta}-1=0.045$

1.6.9. $960e^{2\delta}=K$, $1200e^{\delta}=K$, $\Rightarrow e^{\delta}=\frac{1200}{960}=1.25$. $1-d=e^{-\delta}=0.8$ $d=0.2$

$K=1500$. If $d=0.1$, $pv=1500(1-0.1)^2=1215.$